

HIERARCHICAL MAJORANA SCALES IN THE SEESAW MODEL

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If the mixing angles in each of the seesaw sectors are all small, and the neutrino masses are hierarchical, we study the conditions for large neutrino mixing using triangular matrices. In particular, the implication of the neutrino oscillation data for the mass hierarchy in the heavy Majorana sector is examined, and the heavy Majorana scales are shown to depend sensitively on the solar mixing angle.

1 The Seesaw Mixing Matrix

The neutrino seesaw model¹, $m_{\text{eff}} = -m_D M^{-1} m_D^T$, involves both the Dirac and the Majorana mass matrices of which we have little information. In grand unified theories, the Dirac neutrino mass matrix is often related to the quark mass matrix. A similar analogy does not exist for the heavy Majorana neutrino mass matrix. Here we will try to understand the implications of current neutrino oscillation data^{2,3,4} for the heavy Majorana sector⁵.

There are three components in the seesaw contributing to the effective neutrino mixings: left-handed (LH) rotations in the charged lepton sector (U_l), LH rotations in the Dirac neutrino sector (U_d), and right-handed (RH) mixings in the Dirac and heavy Majorana neutrino sectors. The effective neutrino mixing matrix is defined as $\nu_i = U \nu_\alpha$, with $i = e, \mu, \tau$ and $\alpha = 1, 2, 3$ for the mass eigenstates. We can write $U = U_l^\dagger U_d U_{ss}$, where the seesaw mixing matrix⁶ U_{ss} is due to RH mixings in the Dirac and Majorana sectors:

$$U_{ss}^\dagger m_D^{\text{diag}} V_R M_{\text{diag}}^{-1} V_R^T m_D^{\text{diag}} U_{ss}^* = m_{\text{eff}}^{\text{diag}}. \quad (1)$$

Large neutrino mixing can be due to any of the three components or a certain combination of them^{6,7,8,9,10,5}. Here we will assume $U_l \simeq U_d \simeq 1$ so that we may identify U with U_{ss} and investigate the restrictions on V_R and the heavy Majorana scales by present data. We will consider the limit of hierarchical neutrino masses in all sectors and small RH mixing angles in V_R . For numerical estimates, we neglect possible phases⁷ in the mass matrices and take the Dirac neutrino masses to be the up type quark masses.

Eq. (1) has a quadratic dependence on both V_R and U_{ss} . The analysis can be greatly simplified by reducing Eq. (1) to a linear equation. This is done by writing⁵ $m_{\text{eff}} = N N^T$, with $N = m_D^{\text{diag}} V_R M_{\text{diag}}^{-\frac{1}{2}}$, and dealing with N

instead. Denoting the mass eigenvalues of M by $M_i = R_i^{-2}$, the N matrix is to a good approximation of lower triangular form:

$$N \simeq \begin{pmatrix} R_1 m_1 V_{11} & 0 & 0 \\ R_1 m_2 V_{21} & R_2 m_2 V_{22} & 0 \\ R_1 m_3 V_{31} & R_2 m_3 V_{32} & R_3 m_3 V_{33} \end{pmatrix}, \quad (2)$$

where m_i are the Dirac neutrino masses, and V_{ij} are the elements of the RH mixing V_R . N has a linear dependence on V_R and R_i and will be our center of focus. What do the neutrino data tell us about the N matrix?

2 General Solution

When we write $m_{\text{eff}} = NN^T$, N is defined up to an arbitrary RH rotation $N \sim NO_R$. This ambiguity can be removed by putting N into lower triangular form^{5,11} through a properly chosen O_R .

If $N = U_L \text{diag}(n_1, n_2, n_3) U_R$, the effective neutrino masses are given by n_i^2 and the mixing is $U = U_L$. Thus all the information about effective neutrinos is contained in N . Similarly, N is uniquely determined in its lower triangular form by the masses and mixing angles of the effective neutrinos. The properties of the heavy Majorana sector can then be read off by identifying this triangular N matrix with that of Eq. (2).

The atmospheric neutrino data² suggest that ν_μ - ν_τ oscillation is consistent with maximal mixing. Denoting the solar ν_e - ν_μ mixing angle by θ and ν_e - ν_τ mixing angle by ϵ , the mixing matrix can be written as $U = R_{23}(\pi/4)R_{13}(\epsilon)R_{12}(\theta)$. The angle ϵ is small⁴.

Depending on the size of ϵ , N can have three different patterns, with the largest elements located in the (a) (2,2), (3,2), (b) (i, j) , $i, j = 1, 2$, (c) (2,1), (3,1) positions, respectively. However, only (a) does not involve fine-tuning in the N matrix. Type (a) can be obtained in the limit $n_1/n_2 \ll \theta$ and $\epsilon \ll m_2^{\text{eff}}/m_3^{\text{eff}}$. The N matrix for (a) has a simple lower triangular form⁵:

$$N \simeq \begin{pmatrix} s_\theta n_2 & 0 & 0 \\ \frac{1}{\sqrt{2}}c_\theta n_2 & \frac{1}{\sqrt{2}}n_3 & 0 \\ -\frac{1}{\sqrt{2}}c_\theta n_2 & \frac{1}{\sqrt{2}}n_3 & \sqrt{2}\frac{n_1}{s_\theta} \end{pmatrix}, \quad (3)$$

where $c_\theta = \cos \theta$, and $s_\theta = \sin \theta$.

Comparison between Eq. (2) and Eq. (3) gives the RH mixing angles:

$$\frac{V_{32}}{V_{22}} = \frac{m_c}{m_t}, \quad \frac{V_{21}}{V_{11}} = \frac{m_u}{\sqrt{2} \tan \theta m_c}, \quad \frac{V_{31}}{V_{11}} = \frac{-m_u}{\sqrt{2} \tan \theta m_t}. \quad (4)$$

All three RH angles are thus much smaller than one, consistent with our assumption from the start.

Identifying the diagonal elements of Eqs. (2) and (3) yields three additional relations: $m_3^{\text{eff}} = 2m_c^2/M_2$, $m_2^{\text{eff}} = m_u^2/s_\theta^2 M_1$, and $m_1^{\text{eff}} = s_\theta^2 m_t^2/2M_3$. Note that the heaviest light neutrino is quadratically dependent on m_c rather than m_t . For hierarchical effective neutrino masses, $m_3^{\text{eff}} = \sqrt{\Delta m_{\text{atm}}^2}$ and $m_2^{\text{eff}} = \sqrt{\Delta m_{\text{solar}}^2}$. The three heavy Majorana scales can then be obtained by reversing the above three relations:

$$M_2 \simeq \frac{2m_c^2}{\sqrt{\Delta m_{\text{atm}}^2}} \simeq 6 \times 10^9 \text{ GeV} \quad (5)$$

and

$$\frac{M_1}{M_2} = \frac{m_u^2 m_3^{\text{eff}}}{2m_c^2 s_\theta^2 m_2^{\text{eff}}} , \quad \frac{M_2}{M_3} = \frac{4m_c^2 m_1^{\text{eff}}}{m_t^2 s_\theta^2 m_3^{\text{eff}}} , \quad (6)$$

where we have used $m_c(M_2) \approx 0.4$ GeV, and $\Delta m_{\text{atm}}^2 \approx 3 \times 10^{-3}$ eV². Note that different from M_2 , M_1 and M_3 depend on the solar mixing angle.

3 Hierarchical Majorana Scales

We now examine the hierarchy among the heavy Majorana scales for each of the solar solutions. It is clear from Eq. (6) that whereas M_1 may not be too far below the scale M_2 , M_3 is always far above M_2 independent of the particular solar solution.

3.1 Small angle MSW

Taking³ $\sin^2 2\theta_{\text{solar}} \approx 5 \times 10^{-3}$ and $\Delta m_{\text{solar}}^2 \approx 5 \times 10^{-6}$ eV², we get

$$M_1 \approx 7 \times 10^8 \text{ GeV} \quad M_3 \approx r \cdot 4 \times 10^{12} \text{ GeV} \quad (7)$$

where $r = m_2^{\text{eff}}/m_1^{\text{eff}} > 1$.

3.2 Large angle MSW

Using³ $\sin^2 2\theta_{\text{solar}} \approx 0.8$ and $\Delta m_{\text{solar}}^2 \approx 3 \times 10^{-5}$ eV², we have

$$M_1 \approx 1 \times 10^6 \text{ GeV} \quad M_3 \approx r \cdot 4 \times 10^{14} \text{ GeV} . \quad (8)$$

3.3 Vacuum oscillation

Taking³ $\theta_{\text{solar}} \approx 45^\circ$ and $\Delta m_{\text{solar}}^2 \approx 7 \times 10^{-11}$ eV², we find

$$M_1 \approx 5 \times 10^8 \text{ GeV} \quad M_3 \approx r \cdot 4 \times 10^{17} \text{ GeV} . \quad (9)$$

The scale M_3 being way above M_{GUT} makes the vacuum oscillation solution unlikely to be viable.

For all three solutions, we find a hierarchy among the heavy Majorana scales $M_1 \ll M_2 \ll M_3$, and the separation between scales depends sensitively on the specific solution to the solar neutrino problem. The same conclusion was reached in a different way¹⁰.

In summary, though it is possible to get large neutrino mixing from hierarchical masses and small mixing angles in all of the seesaw components, a strong hierarchy of heavy Majorana scales seems to be necessary.

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